	Centre National d'Etudes Spatiales	
SCA_RAD_11 TRO-34-NT-2801-CNES	**Activity : CAL/VAL** **Scarab** **equations**	**Prepared by :** **A Rosak** **Verified by :** **N Karouche**

Contents

1. OBJECTIVE

The objective of this note is to detailed equations that are used for Scarab.

2. BANDWITH DEFINITION

Canal	Description	Intervalle spectral
1	Visible (VIS)	0,5 — 0,7 µm
2	Solar (SW)	0,2 — 4 µm
3	Total (T)	0,2 — 200 µm
4	IR window (IRW)	10 — 13 µm

3. NOTATION

$R_k(\lambda)$ is the spectral response of the channel k, without solar filter, and not normalized.

$R_{k\_sw}^{with\_filter}(\lambda) = R_{k\_sw}(\lambda)$ is the spectral response of the channel k, with solar filter, and not normalized.

$r_2^{N\_without\_filter}(\lambda)$ is the spectral response of the channel 2, without solar filter, normalized at 310K

$r_3^{N\_without\_filter}(\lambda)$ is the spectral response of the channel 3, without solar filter, normalized at 310K

It is the nominal mode for channel 3.

$r_{2\_sw}^{N}(\lambda)$ is the spectral response of the channel 2, with solar filter, normalized at 5800K

It is the nominal mode for channel 2.

Theses notations are equivalent :

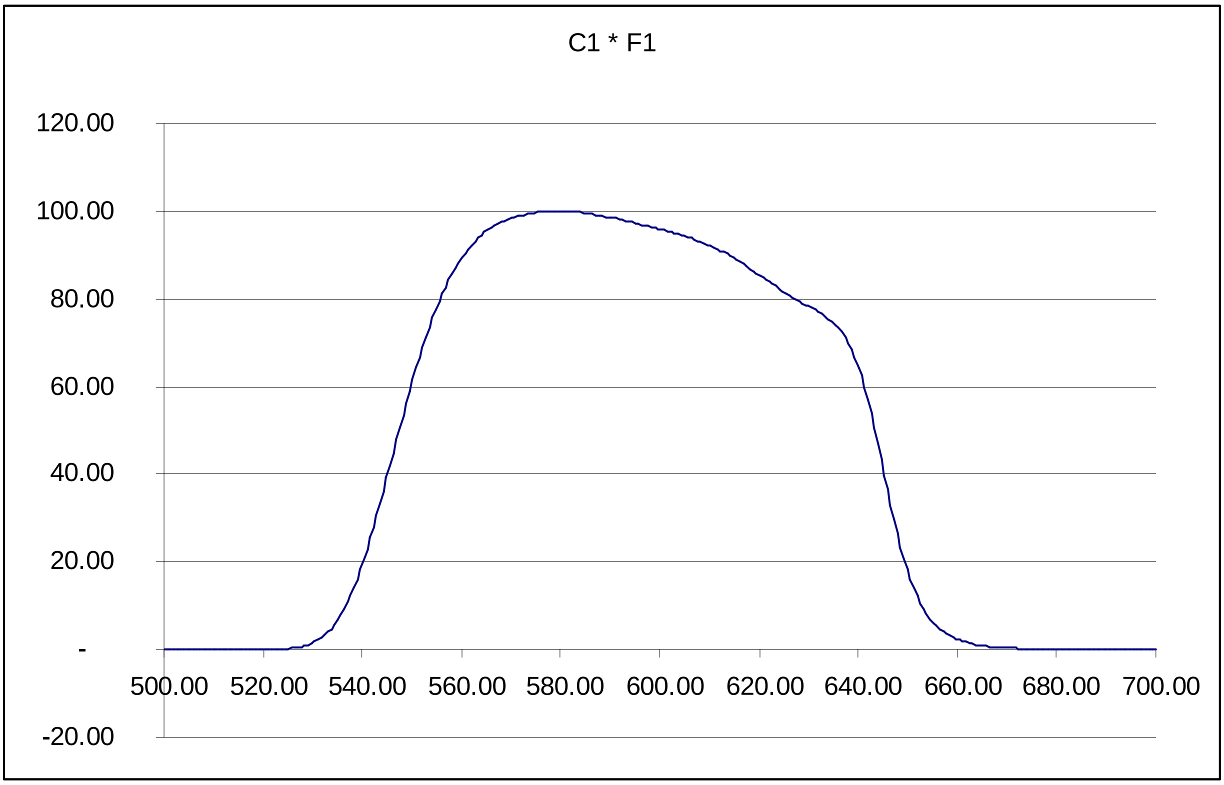
$r_k^N(\lambda) = r_k^{N\_without\_filter}(\lambda)$

$r_{k\_sw}^N(\lambda) = r_{k\_sw}^{N\_with\_filter}(\lambda)$

BB initials, depending of the context, can replace **B**lack **B**ody.

4. CHANNEL 1

Spectral response :



The spectral response is normalized with :

$$r_1^N(\lambda) = R_1(\lambda).\frac{\sum_{0.5\mu m}^{0.7\mu m} L_{corps\_noir}(\lambda, 5800K) * 0.001\mu m.}{\sum_{0.5\mu m}^{0.7\mu m} R_1(\lambda).L_{corps\_noir}(\lambda, 5800K) * 0.001\mu m}$$

The spectral response is given between 500nm and 700 nm, with an increment of 1nm.

Gain

The gain is calculated in theses conditions :

In front of a pure blackbody at 5800K, the channel 1 response is N_1 numerical count. The gain is determined by :

$$G_1 = \frac{\overline{N_1^{BB5800}}}{\int_{\lambda 1}^{\lambda 2} r_1^N(\lambda).L_{BB}(\lambda,5800).d\lambda}$$

$$= \frac{\overline{N_1^{BB5800}}}{\sum_{0.5\mu m}^{0.7\mu m} r_1^N(\lambda).L_{BB}(\lambda,5800).d\lambda}$$

$$= \frac{\overline{N_1^{BB5800}}}{\sum_{0.5\mu m}^{0.7\mu m} L_{BB}(\lambda,5800) * 0.001\mu m}$$

Filtered radiance

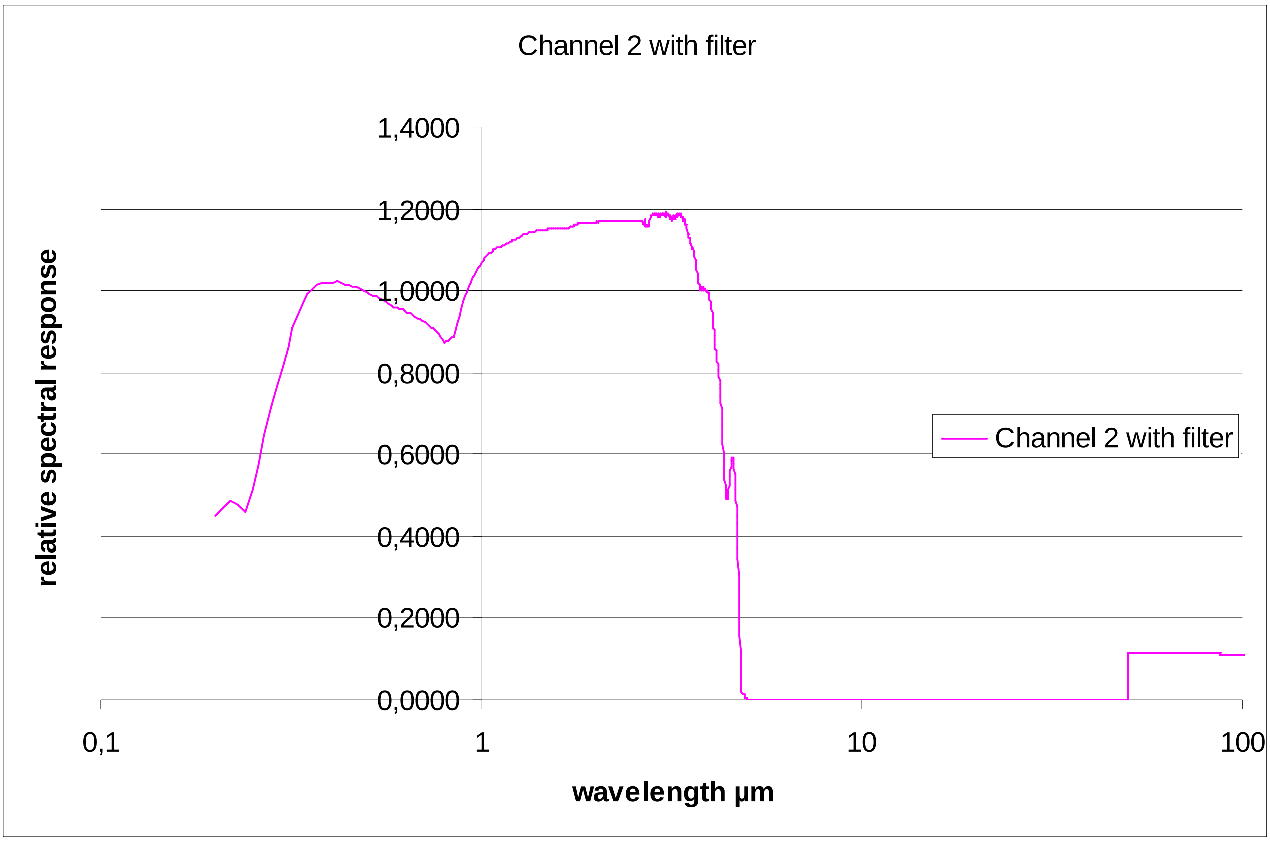
$$L_{1\_earth}^{f} = \sum_{0.5\mu m}^{0.7\mu m} r_1^N(\lambda) * L_{earth}(\lambda) * 0,001\mu m$$

According to the gain definition, in front of a pure black body at 5800K, the filtered radiance measured by channel 1 will be :

$L_{1\_BB5800}^{f} =$ 4 920 700 W/m²/sr

5. CHANNEL 2

Spectral response :



The spectral response is normalized with :

First, we can define the normalized spectral response of channel 2, without filter, at 310K.

$$r_2^N(\lambda) = R_2(\lambda).\frac{\sum_{0.2\mu m}^{200\mu m} L_{BB}(\lambda,310K)*0.01\mu m.}{\sum_{0.2\mu m}^{200\mu m} R_2(\lambda).L_{BB}(\lambda,310K)*0.01\mu m}$$

Then we can define the normalized spectral response of channel 2, with filter, at 5800K.

$$r_{2\_sw}^{N\_5800}(\lambda) = r_{2\_sw}^{N}(\lambda)$$

$$= T_{filter}(\lambda).r_2^N(\lambda)\frac{\int_0^\infty L_{BB}(\lambda,5800K).d\lambda}{\int_0^\infty T_{filter}(\lambda).r_2^N(\lambda).L_{BB}(\lambda,5800K).d\lambda}$$

$$= T_{filter}(\lambda).r_2^N(\lambda)\frac{\sum_{0.2\mu m}^{4\mu m} L_{BB}(\lambda,5800K)*0.01\mu m}{\sum_{0.2\mu m}^{4\mu m} T_{filter}(\lambda).r_2^N(\lambda).L_{BB}(\lambda,5800K)*0.01\mu m}$$

The spectral response is given between 0.2µm and 4µm, with an increment of 10nm.

Gain

The gain is calculated in theses conditions:

In front of a pure blackbody at 5800K, the channel 2 response is N_2 numerical count. The gain is determined by :

$$G_{2\_sw} = \frac{\overline{N_2^{BB5800}}}{\int_{\lambda 1}^{\lambda 2} r_{2\_sw}^{N}(\lambda).L_{BB}(\lambda,5800).d\lambda}$$

$$= \frac{\overline{N_2^{BB5800}}}{\sum_{0.2\mu m}^{4\mu m} r_{2\_sw}^{N}(\lambda).L_{BB}(\lambda,5800).d\lambda}$$

$$= \frac{\overline{N_2^{BB5800}}}{\sum_{0.2\mu m}^{4\mu m} L_{BB}(\lambda,5800).d\lambda}$$

Filtered radiance

$$L^{f}_{2\_sw\_earth} = \sum_{0.2\mu m}^{4\mu m} r_{2\_sw}^{N}(\lambda).L_{earth}(\lambda) * 0.01\mu m$$

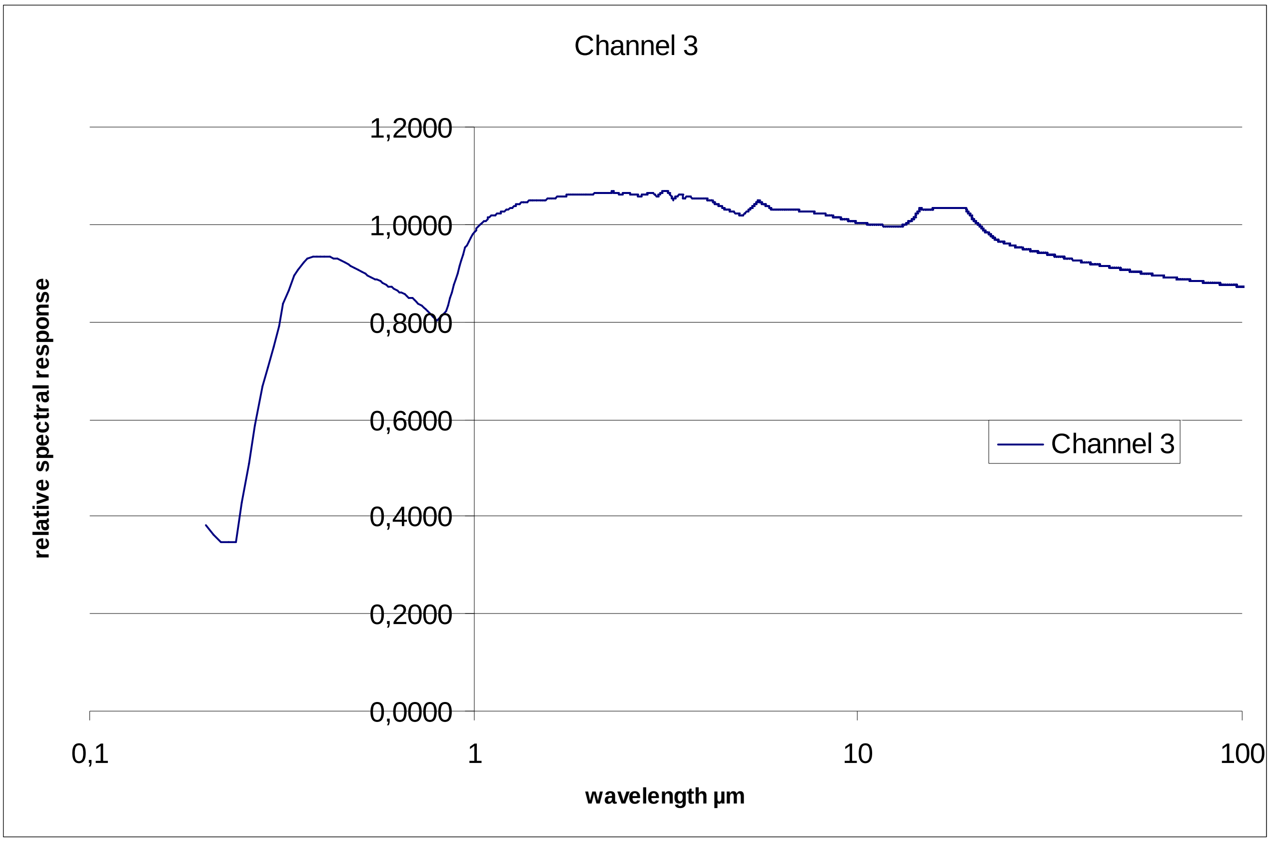
According to the gain definition, in front of a pure black body at 5800K, the filtered radiance measured by channel 2 will be:

$$L^{f}_{2\_SW\_BB5800} = \sum_{0.2\mu m}^{4\mu m} L_{BB}(\lambda,5800) * 0.01\mu m$$

$$= 20\,204\,790 \text{ W/m}^2\text{/sr}$$

6. CHANNEL 3

Spectral response :



The spectral response is normalized with:

r_3^N is the spectral response of channel 3, without filter, normalized at 310K

$$r_3^N(\lambda) = R_3(\lambda).\frac{\sum_{0.2\mu m}^{200\mu m} L_{BB}(\lambda, 310K) * 0.01\mu m}{\sum_{0.2\mu m}^{200\mu m} R_3(\lambda).L_B(\lambda, 310K) * 0.01\mu m}$$

The spectral response is given between 0.2µm and 200µm, with an increment of 10nm.

Gain

The gain is calculated in theses conditions:

In front of a pure blackbody at 310K, the channel 3 response is N_3 numerical count. The gain is determined by:

$$G_3 = \frac{\overline{N_3^{BB310}}}{\int_{\lambda 1}^{\lambda 2} r_3^N(\lambda).L_{BB}(\lambda,310).d\lambda}$$

$$= \frac{\overline{N_3^{BB310}}}{\sum_{0.2\mu m}^{200\mu m} r_3^N(\lambda).L_{BB}(\lambda,310).d\lambda}$$

$$= \frac{\overline{N_3^{BB310}}}{\sum_{0.2\mu m}^{200\mu m} L_{BB}(\lambda,310).d\lambda}$$

Filtered radiance

$$L^f{}_{3\_earth} = \sum_{0.2\mu m}^{200\mu m} r_3^N(\lambda).L_{earth}(\lambda) * 0.01\mu m$$

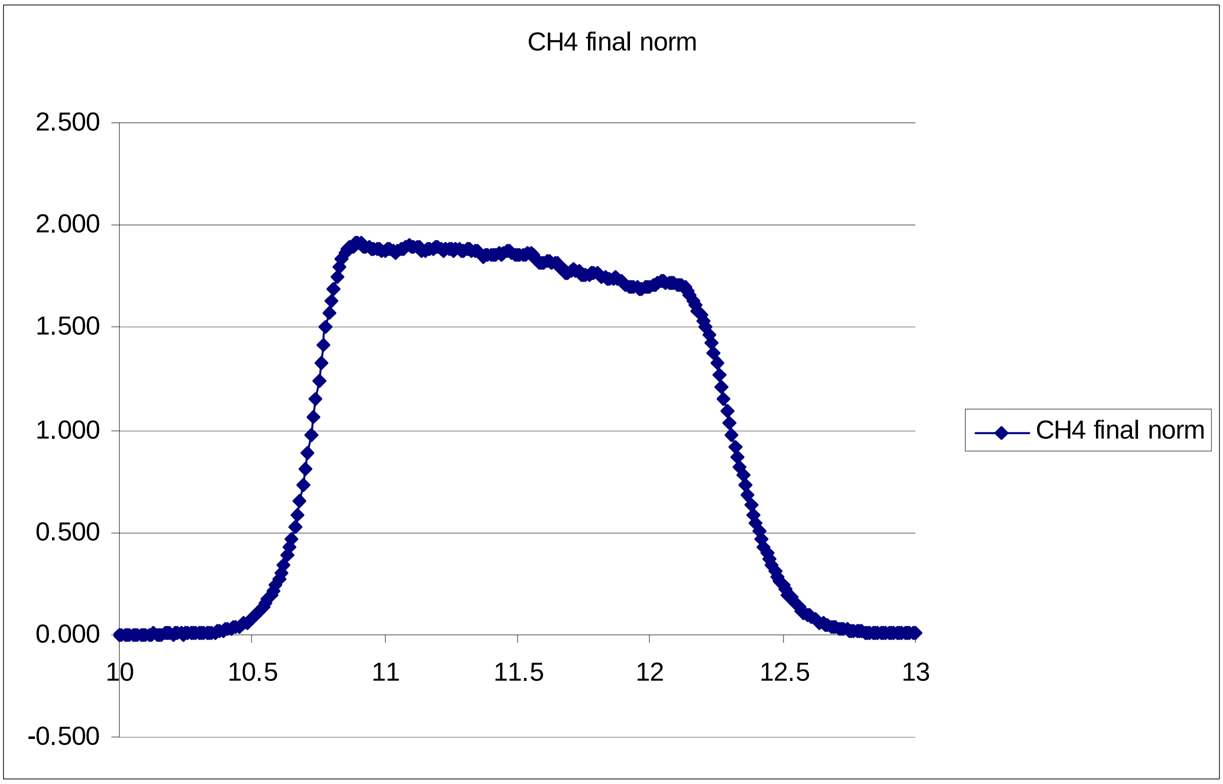
According to the gain definition, in front of a pure black body at 310K, the filtered radiance measured by channel 3 will be:

$$L_3^f = \sum_{0.2\mu m}^{200\mu m} L_{BB}(\lambda,310) * 0.01\mu m$$

$$= 166.59 \text{ W/m}^2\text{/sr}$$

7. CHANNEL 4

Spectral response :



The spectral response is normalized with:

$$r_4^N(\lambda) = R_4(\lambda).\frac{\sum_{10\mu m}^{13\mu m} L_{BB}(\lambda,310K)*0.01\mu m.}{\sum_{10\mu m}^{13\mu m} R_4(\lambda).L_{BB}(\lambda,310K)*0.01\mu m}$$

The spectral response is given between 10µm and 13µm, with an increment of 10nm.

Gain

The gain is calculated in theses conditions:

In front of a pure blackbody at 310K, the channel 4 response is N_4 numerical count. The gain is determined by :

$$G_4 = \frac{\overline{N_4^{BB310}}}{\int_{\lambda 1}^{\lambda 2} r_4^N(\lambda).L_{BB}(\lambda,310).d\lambda}$$

$$= \frac{\overline{N_4^{BB310}}}{\sum_{10\mu m}^{13\mu m} r_4^N(\lambda).L_{BB}(\lambda,310).d\lambda}$$

$$= \frac{\overline{N_4^{BB310}}}{\sum_{10\mu m}^{13\mu m} L_{BB}(\lambda,310) * 0.01\mu m}$$

Filtered radiance

$$L^f{}_{4\_earth} = \sum_{10\mu m}^{13\mu m} r_4^N(\lambda).L_{earth}(\lambda) * 0.01\mu m$$

According to the gain definition, in front of a pure black body at 310K, the filtered radiance measured by channel 4 will be:

$$L^f_{4\_BB310} = \sum_{10\mu m}^{13\mu m} L_{BB}(\lambda,310) * 0.01\mu m$$

$$= 31.49 \text{ W/m}^2\text{/sr}$$

8. ANNEX

The equation used in this document to compute the radiance of the black body is:

$$L_{BB}(\lambda,310K) = \frac{2hc^2}{\lambda^5}.\frac{1}{e^{hc/\lambda k_b T} - 1} = \frac{c1(10^4)^4}{\lambda^5}.\frac{1}{e^{C2*1e4/\lambda T} - 1} \qquad \lambda \text{ is in } \mu\text{m}$$

L unit is W/m²/sr/µm

- c = 299 792 458 m/
- h = 6,626 17×10^{-34} J.s
- k = 1,380 66×10^{-23} J/K

C1=1.191 10^{-8}

C2=1.439